Consequences of the Cauchy Integral formula



Giacinto Morera

1) Moreva Theorem.

Let f be continuous in a region A.

Assume that for any 20 c 1 FB (20,1) C 1 in following property:

if dR < 13 (20,1) - rectangle then \$ f(2) dz=0.

Then $f \in \mathcal{A}(\Omega)$.

Proof. Fix z. e.M. We know that 3 FeA

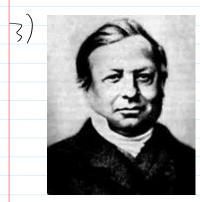
such that Fix=f(z) Yz e B(z.,r). But F

ditterentiable => f'(z) exists.

It is true for any zo => fex(2) =

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Then
$$|f^{(n)}(z_0,R)|$$
, $|f(z)| \leq M \quad \forall z \in B(z_0,R)$
 $\frac{Then}{Then} \quad |f^{(n)}(z_0)| \leq M \quad \text{and} \quad R^{-n}$
 $\frac{Proof.}{f^{(n)}(z_0)} = \frac{N!}{(z_0-z_0)^{n+1}} \int_{C}^{R} \frac{f^{(n)}(z_0)}{(z_0-z_0)^{n+1}} \int_{C}^{R} \frac{f^{(n)}(z_0)}{(z_0-z_0$



Joseph Liouville

Theorem (Liouville) Any bounded
is identically constant.

Reminder tis called entire if the Proof. Left to be beaunder of

Thus $3R: \left| \frac{1}{p(z)} \right| < 1 \forall z : |z| > R$.

Let $M = \max_{|z| \in R} \frac{1}{|p(z)|}$. Then $\forall z$.

So, by Lion ville, $\frac{1}{p(z)} = const - co$



Theorem (Weierstras Let (fn) be a segni in a region A. As

Karl Weierstrass

& + d== 1, m & +n d== 0. By M

N→∞ 8 +n d== 0. By M

2) Fix $z_0 \in \Omega$. Take v: 2Then $B(z_0, 2r) \subset \Omega$, $f_n = f$ unitormly on $B(z_0, r)$, le

Then, by Canchy inequalities, $|f_n^{(k)}(z) - f_n^{(k)}| = |(f_n - f)^{(k)}(z)| \leq \kappa!$ $|f_n^{(k)}(z)| = |(f_n - f)^{(k)}(z)| \leq \kappa!$ $|f_n^{(k)}(z)| = |f_n^{(k)}(z)| \leq \kappa!$ $|f_n^{(k)}(z)| = |f_n^{(k)}(z)| \leq \kappa!$

Restatement; $f_n \in A(\Omega)$, $\sum_{h=1}^{\infty} f_n$ Then $f \in A(\Omega)$ and $\forall \kappa : \sum_{h=1}^{\infty} f_n \in A(\Omega)$